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TOP QUARK PAIR PRODUCTION IN e^+e^- ANNIHILATION NEAR THRESHOLD*

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Recent progress in calculations of the total cross section for top quark pair production near threshold is reviewed. Different top quark mass definitions adequate for threshold studies are discussed. A relation between the potential subtracted mass and the 1S mass is studied. The potential subtracted 1S mass is defined which incorporates attractive features of both schemes.

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1. Introduction

Recently an impressive progress has been achieved in calculations of cross sections for top quark pair production in e^+e^- annihilation near threshold. A future linear collider (LC) operating at energies close to $t\bar{t}$ threshold will be an ideal machine to study properties of the top quark. Prospects that LC will be built during the next decade stimulate growing interest in precise theoretical description of this reaction. In this article I concentrate only on new developments in the years 1998-99. Older calculations are described in reviews, see *e.g.* [1–7] and references cited therein. In Sec. 2 a considerable increase of precision is described due to new mass definitions [8, 9] which are more adequate than the pole mass [10, 11] for threshold studies of the total cross section. The potential subtracted 1S mass is proposed as a combination of the potential subtracted [8] and the 1S [9] mass definitions for the top quark. In Sec. 3 a brief review of recent calculations of higher order corrections is presented.

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2. Mass definitions

2.1. Pole mass and potential subtracted mass

One of the main goals of top quark physics at LC is a precise determination of top quark mass. Expected luminosities and beam energy resolutions are so good that a measurement of this mass with precision better than 100 MeV is conceivable from experimental point of view. Even better precision of theoretical studies is therefore mandatory. At present there are a few sources of theoretical uncertainties. One of them, relativistic corrections are considered in the following section. In this section we use non-relativistic approximation and consider the top quark as a stable particle characterized by the pole mass $m_{\rm pole}$. In this approximation interactions between t and $\bar{t}$ are described by an instantaneous chromostatic potential which in momentum space is conventionally written as

$$V(q) = -C_F \frac{4\pi\alpha_V(q)}{q^2}\,, \tag{1}$$

where $q^2 = |\boldsymbol{q}|^2$ denotes the square of (three)momentum transfer $\boldsymbol{q}$ and $C_F = 4/3$. This formula looks quite similar to the well known Coulomb potential. However, for our purposes we need a better precision and cannot neglect q dependence of the function α_V. In QCD the coupling α_V is running and at present its evolution is known up to two-loop accuracy [12, 13] in perturbative calculations. The coupling α_V can be expressed in terms of the conventional strong coupling constant $\alpha_{\overline{MS}}$ and the relation including terms $\alpha^3_{\overline{MS}}$ has been derived in [12]. The coupling $\alpha_{\overline{MS}}(q)$ is also running and the first four coefficients $(\beta_0, \ldots, \beta_3)$ are known for its renormalization group β function. Two-loop accuracy means that all these coefficients are included in the renormalization group equation for $\alpha_{\overline{MS}}$ and α_V is calculated including also terms $\alpha^3_{\overline{MS}}$. We shall also use α_V calculated in one-loop accuracy, *i.e.* including one order less in the relation between α_V and $\alpha_{\overline{MS}}$ as well as in the renormalization group equation for $\alpha_{\overline{MS}}(q)$. Conventionally $\alpha_{\overline{MS}}(M_Z)$, *i.e.* the value of the strong coupling constant at Z^0 peak is used as a starting point for the evolution.

The coupling $\alpha_V(q)$ grows with decreasing q and around 1 GeV becomes comparable to or larger than 1, and eventually at some point even infinite. In this range of q we cannot trust perturbative expansions and are forced to use some non-perturbative methods or extra phenomenological input to calculate the potential $V(q)$. Unfortunately the Lippmann-Schwinger equation for the energy levels of $t\bar{t}$ system contains an integral over momentum transfers including the dangerous region of low q. Therefore, we have to estimate how much are the energies of toponium states affected by contributions from this region. In other words we have to estimate theoretical uncertainties due to present poor knowledge of the non-perturbative QCD potential. Let q_m be a momentum transfer such that for $q > q_m$ a perturbative formula $V_{pert}(q)$ is sufficiently accurate whereas for $q < q_m$ some non-perturbative expression should be used. In the following discussion we assume that $m_{\rm pole} = 175$ GeV and $q_m = 3$ GeV. As the non-perturbative potential we choose the one proposed by Richardson [14]. Richardson potential depends on a non-perturbative parameter $\Lambda_{\rm R}$ which after Fourier transformation to the position space can be determined from the slope of the linear confining potential. A successful description of $b\bar{b}$ and $c\bar{c}$ nS states is obtained for $\Lambda_{\rm R} = 0.4$ GeV. A formula for the Richardson potential as well as a description of its numerical implementation are given in [15], see Appendix A therein. For the perturbative part of the potential we use $\alpha_V(q)$ calculated with one-loop accuracy and $\alpha_{\overline{MS}}(M_Z) = 0.118$. In the pole mass scheme the binding energies of toponium states are defined as

$$E_r^{\rm pole} = M_r - 2m_{\rm pole}\,, \tag{2}$$

where M_r denotes the rest mass of the state r. Thus $E_{1S}^{\rm pole}$ is the binding energy of the $1S$ state.

In Table I the values of $E_{1S}^{\rm pole}$ are given for a few values of $\Lambda_{\rm R}$. As already explained the realistic values are obtained for $\Lambda_{\rm R}$ around 0.4 GeV. In the

TABLE I

Binding energies and energy shifts for toponium resonances in the pole mass and in the potential subtracted mass scheme for different values of $\Lambda_{\rm R}$ and $\mu_f = 5$ GeV.

$\Lambda_{\rm R}$	$E_{1S}^{\rm pole}$	$2\delta m(\mu_f)$	$E_{1S}^{\rm PS}(\mu_f)$	$E_{2S}^{\rm PS}(\mu_f)$	$E_{3S}^{\rm PS}(\mu_f)$	$E_{4S}^{\rm PS}(\mu_f)$
0.01	– 2.273	0.827	– 1.446	0.152	0.559	0.668
0.1	– 2.616	1.171	– 1.445	0.163	0.602	0.786
0.2	– 2.785	1.340	– 1.445	0.171	0.629	0.848
0.4	– 2.956	1.511	– 1.445	0.184	0.679	0.949
0.6	– 3.014	1.567	– 1.447	0.197	0.725	1.042
1.0	– 2.928	1.474	– 1.454	0.228	0.828	1.234

range 0.2–0.6 GeV the variation of $E_{1S}^{\rm pole}$ is reasonably moderate and one can conclude that determination of $m_{\rm pole}$ from a measurement of 1S state mass M_{1S} is possible with theoretical uncertainty of order 100 MeV due to contributions from the non-perturbative region. Even for a very drastic change of the phenomenological potential and $\Lambda_{\rm R} = 0.01$ GeV the change in $m_{\rm pole}$ for fixed M_{1S} is about 350 MeV. All this means that 1S toponium state is too small to be affected significantly by momentum transfers below q_m. In fact the situation is much better than it follows from a moderate

dependence of $E_{1S}^{\rm pole}$ on $\Lambda_{\rm R}$. The latter is due to a dependence of $m_{\rm pole}$ on small momentum transfers. Beneke proposed [8] to replace $m_{\rm pole}$ by the potential subtracted (PS) mass

$$m_{\rm PS}(\mu_f) = m_{\rm pole} - \delta m(\mu_f)\,, \tag{3}$$

where

$$\delta m(\mu_f) = -\frac{1}{2}\int\limits_{q<\mu_f} \frac{d^3\boldsymbol{q}}{(2\pi)^3} V(\boldsymbol{q})\,. \tag{4}$$

In Eq. (4) μ_f is an arbitrary parameter larger than q_m, *i.e.* μ_f should be chosen in the region of momentum transfers where the perturbative expansion is sufficiently accurate. Furthermore it can be demonstrated that such a definition corresponds to a mass parameter which is not sensitive to small momentum transfers [8]. Let us define the energy shift[1] for a state r in the PS scheme as

$$E_r^{\rm PS}(\mu_f) = M_r - 2m_{\rm PS}(\mu_f)\,. \tag{5}$$

It follows that

$$E_r^{\rm PS}(\mu_f) = E_r^{\rm pole} + 2\delta m(\mu_f)\,. \tag{6}$$

In Table I the values of $2\delta m(\mu_f)$ are given for $\mu_f = 5$ GeV and a few values of $\Lambda_{\rm R}$. As expected this quantity also changes with $\Lambda_{\rm R}$. It is remarkable, however, that the variations of $2\delta m(\mu_f)$ and $E_{1S}^{\rm pole}$ cancel each other and the energy shift for 1S bound state becomes surprisingly stable, see Table I. The energy shifts for nS bound states up to n=4 are also given in Table I. It is seen that the dependence on $\Lambda_{\rm R}$ is reasonably small for 2S state. However for 3S and 4S a significant dependence on $\Lambda_{\rm R}$ persists which means that these radial excitations are spatially large enough to be affected by low momentum transfers.

Coming back to 1S state we observe that the precision which can be achieved in determination of $m_{\rm PS}$ is dominated by the measurement of M_{1S}, *cf.* Eq. (5). Contrary to a widespread belief in this case the large width of the top quark does not help at all by cutting off non-perturbative dynamics at low momentum transfers and large spatial distances. In the real world the toponium 1S resonance has the width of about 3 GeV which is a large number when compared to 100 MeV precision to be achieved in determination of the top quark mass. Of course, for larger energies, say 2 GeV or more above 1S level the top width helps. However, for a measurement located in energy close to 1S state a real problem is how to unravel its effects.

[1] 'Binding energy' would be a misleading terminology because, depending on the values of μ_f the energy shifts for some or even all bound states can be positive.

2.2. Potential subtracted 1S mass

Recently Hoang and Teubner have observed [9] that it is very convenient to perform calculations in a scheme in which the mass of the top quark is defined simply as one half of M_{1S} in the limit of zero top width. They proposed the name 1S mass for such mass parameter and demonstrated that it is a short distance one, *i.e.* unlike the pole mass is not sensitive to dynamics at large distances. I think that it is useful to consider their proposal as a condition on the mass parameter μ_f in PS scheme. In fact it is straightforward to find μ_{1S} such that

$$E_{1S}^{\rm PS}(\mu_{1S}) = 0\,. \tag{7}$$

One can also show that μ_{1S} is in the perturbative regime and corresponds to a typical momentum transfer for 1S bound state. The corresponding potential subtracted mass

$$m_{P1S} = m_{\rm PS}(\mu_{1S}) = \tfrac{1}{2} M_{1S} \tag{8}$$

is by definition a 1S mass. It is also clear why 1S mass is a better scheme than 2S or 3S mass schemes. The energies of higher radial excitations are simply more affected by non-perturbative small momentum transfers.

A very good stability of $E_{1S}^{\rm PS}(\mu_f = 5\,{\rm GeV})$ guarantees that the mass parameter μ_{1S} does not depend on non-perturbative parameters like $\Lambda_{\rm R}$. On the other hand it depends on the dynamics in the perturbative regime. In particular μ_{1S} depends on the value of $\alpha_{\overline{MS}}(M_Z)$ and on the order of perturbative calculations. For example: at one-loop accuracy and for $\alpha_{\overline{MS}}(M_Z)$= 0.15, 0.18 and 0.21 the corresponding values of μ_{1S} are equal to 13.26 GeV, 13.63 GeV and 14.00 GeV, respectively.

2.3. Remarks

It is evident that the short distance masses discussed in this section are superior and more convenient than the pole mass in studies of the total cross section near threshold. Does it mean that the pole mass is a totally useless concept which should be abandoned for permanently confined quarks? I believe that the answer to this question will be no. I do so because there are other cross sections and distributions which can be measured in experimental studies near threshold. They are less inclusive than the total annihilation cross section and in consequence more difficult from theoretical point of view. It is not precluded that the pole mass can be a good parameter to describe some of them. For example: it is plausible that the invariant mass distribution of top quark decay products has a maximum close to the pole mass rather than to 1S mass. Of course the corresponding mass parameter can be extracted with a limited accuracy. At some level of precision it will be necessary to decide if a pion, which is slow in $t\bar{t}$ center of mass, belongs to decay products of t or to decay products of $\bar{t}$ and this may be impossible even in principle.

3. Top width and higher order corrections

For center-of-mass energies close to the $t\bar{t}$ threshold the top quarks are produced with nonrelativistic velocities $v \ll 1$. Therefore nonrelativistic approximation is a good starting point. However a high precision determination of the top quark mass requires a systematic study of higher order corrections including relativistic and radiative corrections. In comparison to bound state problems like spectroscopy of positronium or hydrogen-like ions, which have been studied in QED, a novel feature of $t\bar{t}$ production near threshold is a very large width of this system. In their pioneering work Fadin and Khoze [16] showed how to incorporate the top width into theoretical descriptions. They proposed to use Green function rather than binding energies and wave functions for individual resonances. Their Leading Order approach (LO) was further developed in [17–19]. In particular QCD static potential was included at one-loop accuracy level and Next-to-Leading Order (NLO) contributions in nonrelativistic expansion were calculated to forward–backward asymmetry [20] and top quark polarization [21, 22]. These early studies were done in the pole mass scheme, so a considerably better accuracy can be obtained by using one of the short distance masses discussed in Sec. 2. Further progress cannot be achieved without performing calculations at Next-to-Next-to-Leading Order (NNLO) including corrections of order v^2, $\alpha_s v$ and α_s^2. This problem is very complicated because in calculations of Green function relativistic and radiative corrections do not factorize and have to be consider simultaneously.

During last two years a number of papers appeared presenting calculations at NNLO level and using completely different techniques [9,23–31]. In particular in [9, 28–31] a complete NNLO results are presented. Qualitatively all these calculations agree quite well. NNLO corrections produce an important shift in the binding energies, *i.e.* in the position of the threshold, and a significant increase of the normalization for the total cross section. However, a rather large uncertainty remains in the normalization due to scale dependence in NNLO corrections. At a more quantitative level a detailed comparison is difficult because different mass definitions are used by different groups.

An important new theoretical development is the so-called Potential Non-Relativistic QCD [32]. In this framework a systematic study of QCD potential in even higher orders can be accomplished. In particular calculations of quarkonium spectrum at order $\alpha_s^5 \ln \alpha_s$ have been presented [32].

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